The difference between the results of using (4.3) and (4.4) for $0 < \gamma < \infty$ evidently does not exceed the difference for $\gamma = \infty$. Results of calculations using (4.3) (solid lines) and (4.4) (dashed lines) are presented in Fig. 2 for $\gamma = 6$, $\gamma = \infty$.

Results of calculating the forces and moments by means of (1.1)-(1.3) (solid lines) and (3.1) and (2.5) (dashed lines) are presented in Figs. 3 and 4 for a shell element deformed according to the law

$$\dot{\varepsilon}_{12} = \dot{\varkappa}_{12} = \dot{\varkappa}_{22} = 0, \quad \dot{\varkappa}_{11} = \chi\dot{\varepsilon}_{22}, \quad \dot{\varepsilon}_{11} = -(1/2)\,\dot{\varepsilon}_{22},$$
$$\dot{\varepsilon}_{22} = 1, \quad 0 < t \leqslant t_1; \quad \dot{\varepsilon}_{22} = -1, \quad t_1 < t \leqslant 2t_1,$$

where the point denotes differentiation with respect to t. The results in Fig. 3 correspond to $\chi = 0.5$ and in Fig. 4 to $\chi = 5$. The calculation was performed for $\nu = 0.3$, $\gamma = 6$, $t_1 = 2$. In evaluating $t_{\alpha\beta}$, $m_{\alpha\beta}$ by means of (1.1)-(1.3) the integrals were replaced by Simpson quadratures with 21 sites. The computation procedure is analogous to that elucidated in [3].

The results in Figs. 2-4, the calculations for other shell element strain paths, and the comparison with the results in [3] all show that (3.1) and (2.5) correspond satisfactorily to (1.1)-(1.3), the difference in the results for $\gamma < \infty$ not exceeding this difference in the case of ideal elastoplastic shell strain [3].

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SINGULAR SOLUTIONS OF EQUATIONS OF SHALLOW SHELLS FOR A CONCENTRATED TANGENTIAL LOAD

V. P. Ol'shanskii

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As we know [1, 2], in the case of action of concentrated loads the solutions of the shell equations have a singular character. These solutions have been set up by various methods primarily for a normal concentrated force. An attempt to obtain the fundamental solutions for a tangential force lead to very cumbersome results [3]. Below, by the method of Fourier integral transforms, it was possible to obtain more compact solutions in the form of power and trigonometric series. As an addition to the well-known results in the analysis of singularities of the stress state in the vicinity of a concentrated source of radius r, it is shown that in addition to the tangential forces increasing as r^{-1} for $r \to 0$, one of the shear forces also has a weaker singularity of logarithmic form. Asymptotic expressions of the behavior of the fundamental solutions for small values of the argument are given.

The analysis of the elastic local stress state is carried out on the basis of the equations of the theory of thin, shallow, isotropic shells. The solution of these equations by means of the two-dimensional Fourier transform, which is expounded in detail in [3], gives the following values of the components of internal force quantities:

$$t_1 = \frac{-2X}{1-\nu}\left[\frac{1-\nu}{2}A_1 + \frac{2-\nu-\nu^2}{2}A_2 + \left(a_1 + \frac{\lambda+\nu}{1-\nu^2}b^4a_5\right)A_5 + \right.$$
$$\left. + \left(a_2 - \nu a_3 + \frac{\lambda+\nu}{1-\nu^2}b^4a_4\right)A_6\right], \quad t_2 = \frac{-2X}{1-\nu}\left[\nu\frac{1-\nu}{2}A_1 + \frac{\nu-1}{2}A_2 + \right.$$
$$\left. + \left(\nu a_1 + \frac{1+\lambda\nu}{1-\nu^2}b^4a_5\right)A_5 + \left(\nu a_2 - a_3 + \frac{1+\lambda\nu}{1-\nu^2}b^4a_4\right)A_6\right],$$

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$$t_{12}=-X[A_3-\nu A_4+(a_1-a_3)A_8+a_2A_7],\quad m_1=\frac{2X}{(1-\nu)R_2}[a_5B_1+\nu a_4B_2+$$

$$+(a_4+\nu a_5)B_3],\quad m_2=\frac{2X}{(1-\nu)R_2}[\nu a_5B_1+a_4B_2+(a_5+\nu a_4)B_3],$$

$$m_{12}=\frac{2X}{R_2}(a_4B_4+a_5B_5), \tag{1}$$

$$q_1=\frac{-2X}{(1-\nu)R_2}(a_4B_7+a_5B_6),\quad q_2=\frac{-2X}{(1-\nu)R_2}(a_4B_8+a_5B_9),$$

where t_1, t_2, t_{12} are tangential and shear forces; m_1, m_2, m_{12} are the bending and twisting moments; q_1, q_2 are the shear forces; X is the external tangential force directed along the line of the principal curvature of the larger radius; R_1, R_2 are the radii of curvature ($R_2 \leq R_1$); ν is Poisson's ratio of the shell material; and h is its thickness. The coefficients $a_{1...5}$ and b^4 are expressed in terms of the shell parameters by the expressions

$$a_1=6(1-\nu)(1+2\lambda\nu+\lambda^2)h^{-2}R_2^{-2},\quad a_2=12\lambda^2(1-\nu^2)h^{-2}R_2^{-2},$$

$$a_3=12\left[\frac{1+\nu}{2}(1+2\lambda\nu+\lambda^2)-(\lambda+\nu)(1+\lambda\nu)\right]h^{-2}R_2^{-2}, \tag{2}$$

$$a_4=\frac{1+\nu}{2}(1+\lambda\nu)-(\lambda+\nu),\quad a_5=-(1-\nu)(\lambda+\nu)2^{-1},$$

$$\lambda=R_2R_1^{-1},\quad b^4=12(1-\nu^2)h^{-2}R_2^{-2},$$

and $A_{1...8}$, $B_{1...9}$ denote two-dimensional Fourier integrals. By means of the Kronecker symbol δ_{jm} they can be written in the form

$$A_j=\frac{i}{4\pi^2}\int\int_{-\infty}^{\infty}[(\xi^2+\eta^2)^4+b^4(\xi^2+\lambda\eta^2)^2]^{-1}\{[\delta_{1j}\xi^3+\delta_{2j}\xi\eta^2+\delta_{3j}\eta^3+$$

$$+\delta_{4j}\eta\xi^2](\xi^2+\eta^2)^2+\delta_{5j}\xi^3+\delta_{6j}\xi\eta^2+\delta_{7j}\eta^3+\delta_{8j}\eta\xi^2\}e^{-i(\xi x+\eta y)}d\xi d\eta,\qquad i=\sqrt{-1}, \tag{3}$$

$$B_j=\frac{i}{4\pi^2}\int\int_{-\infty}^{\infty}[(\xi^2+\eta^2)^4+b^4(\xi^2+\lambda\eta^2)^2]^{-1}\{\delta_{1j}\xi^5+\delta_{2j}\xi\eta^4+\delta_{3j}\xi^3\eta^2+$$

$$+\delta_{4j}\xi^2\eta^3+\delta_{5j}\xi^4\eta-i(\xi^2+\eta^2)[\delta_{6j}\xi^4+\delta_{7j}\xi^2\eta^2+\delta_{8j}\xi\eta^3+\delta_{9j}\xi^3\eta]\}e^{-i(\xi x+\eta y)}d\xi d\eta.$$

Here 0x, 0y are the axes of a rectangular system oriented so that the force X is directed along 0x and the point of its application lies at the origin of the coordinates.

Thus, the investigation of the local stress state of the shell reduces to the analysis of the expressions (3).

In the following we confine ourselves to the case of shells of zero and positive Gaussian curvature for which $0 \leq \lambda \leq 1$. We calculate the integral

$$A_1(x,y)=\frac{1}{4\pi^2}\int\int_{-\infty}^{\infty}\frac{\xi^3(\xi^2+\eta^2)^2\sin\xi x\cdot\cos\eta y}{(\xi^2+\eta^2)^4+b^4(\xi^2+\lambda\eta^2)^2}d\xi d\eta. \tag{4}$$

We go over to the new variables $\xi=\gamma\cos\varphi$, $\eta=\gamma\sin\varphi$, $x=r\cos\varphi$, $y=r\sin\varphi$, having replaced in (4) the product of the sine and cosine by the trigonometric series

$$\sin\xi x\cdot\cos\eta y=2\sum_{k=0}^{\infty}(-1)^kJ_{2k+1}(\gamma r)\cos(2k+1)\theta\cdot\cos(2k+1)\varphi,$$

in which $J_{2k+1}(z)$ is the Bessel function of the first kind of order $2k+1$. We then obtain

$$A_1(r,\theta)=2\pi^{-2}\sum_{k=0}^{\infty}(-1)^k\cos(2k+1)\theta\int_0^{\pi/2}\int_0^{\infty}[\gamma^4+b^4(\cos^2\varphi+\lambda\sin^2\varphi)^2]^{-1}\gamma^4J_{2k+1}(\gamma r)\cos^3\varphi\cdot\cos(2k+1)\varphi d\gamma d\varphi. \tag{5}$$

For the calculation of the inner integral with respect to γ we use the Mellin–Burns representation of Bessel functions [4]:

$$J_\nu(z)=\frac{1}{4\pi i}\int_{c-i\infty}^{c+i\infty}\left(\frac{z}{2}\right)^{-s}\frac{\Gamma\left(\frac{\nu+s}{2}\right)}{\Gamma\left(1-\frac{\nu-s}{2}\right)}ds,\qquad -\nu<c<1.$$

We have

$$L=\int_0^{\infty}\frac{\gamma^4 J_{2k+1}(\gamma r)}{\gamma^4+b_2^4}\,d\gamma=\frac{1}{4\pi i}\int_{c-i\infty}^{c+i\infty}\frac{\Gamma\left(\frac{2k+1+s}{2}\right)}{\Gamma\left(1-\frac{2k+1-s}{2}\right)}\int_0^{\infty}\frac{(\gamma r)^{-s}}{\gamma^4+b_2^4}\,d\gamma ds.$$

Here $b_2^4=b^4(\cos^2\varphi+\lambda\sin^2\varphi)^2$; Γ is the gamma function. Taking into account the fact that [5]

$$\int_0^{\infty}\frac{x^{\rho-1}dx}{(p+qx^{\nu})^{\mu+1}}=\frac{1}{\nu p^{\mu+1}}\left(\frac{p}{q}\right)^{\rho/\nu}\frac{\Gamma(\rho/\nu)\,\Gamma(1+\mu-\rho/\nu)}{\Gamma(1+\mu)},$$
$$0<\rho/\nu<\mu+1,$$

the calculation of L is reduced to the integration on the complex plane along a straight line parallel to the imaginary axis. This is easily realized, since the residues of gamma functions are located on the negative real half-axis. According to the theory of residues, we find

$$L=\frac{1}{r}\sum_{m=0}^{k}\frac{(-1)^m\,\Gamma(1+k-m)\left(\frac{rb_2}{2}\right)^{2m}}{\Gamma(m+k+1)}\cos\frac{m\pi}{2}+\frac{1}{r}\lim_{\mu\to 0}\frac{\pi}{\sin(k-\mu)\pi}\times$$
$$\times\sum_{m=0}^{\infty}\left[\frac{\Gamma(m+k+2+\mu)\left(\frac{rb_2}{2}\right)^{2m+2k+2}}{\Gamma(m+2k+2+\mu)\,\Gamma(m+1+\mu)}\,\frac{\cos\frac{1+m+k}{2}\pi}{\Gamma(m+k+2)}-\frac{\Gamma(m+k+2)\left(\frac{rb_2}{2}\right)^{2+2m+2k-2\mu}\cos\frac{1+m+k-\mu}{2}\pi}{\Gamma(m+k+2-\mu)\,\Gamma(m+2k+2)\,\Gamma(m+1)}\right].$$

Having substituted the value of L into (5), we carry out integration with respect to φ. This is realized by means of the expression

$$\int_0^{\pi}(1-\varepsilon\cos\varphi)^q\cos k\varphi d\varphi=\frac{\pi\Gamma(q+1)}{\Gamma(k+1)\,\Gamma(q-k+1)}\left(\frac{2z}{1+z}\right)^{-q}\left(\frac{z-1}{z+1}\right)^{k/2}{}_2F_1\left(k-q,\,-q;\;k+1;\;\frac{z-1}{z+1}\right),$$

where $z=(1-\varepsilon^2)^{-1/2}$; ${}_2F_1$ is a hypergeometric function. Having taken further the limit with respect to μ, we obtain the final result for A_1. In an analogous manner we calculate the remaining integrals. Omitting the details, we present the results of the calculations:

$$\sum_{s=1}^{8}A_s\delta_{sj}=\frac{1}{4\pi^2}\sum_{k=0}^{\infty}(-1)^k[(1+2\delta_{1357j})\,T_{kj}(k,r)+(3\delta_{15j}-3\delta_{37j}+$$
$$+\delta_{26j}-\delta_{48j})\,T_{kj}(k+1,r)+(1-2\delta_{2468j})\,T_{kj}(k+2,r)+(1-\delta_{2367j})\times$$
$$\times T_{kj}(|k-1|,r)](C_k\delta_{1256j}+S_k\delta_{3478j}),\qquad j=1,2\ldots 8; \tag{6}$$
$$\sum_{s=1}^{9}B_s\delta_{sj}=\frac{1}{128\pi^2}\sum_{k=0}^{\infty}(-1)^k[(\delta_{124j}-\delta_{35j})\,\Phi_{kj}(k+3,r)+$$
$$+(5\delta_{1j}-3\delta_{25j}-\delta_{3479j}+\delta_{68j})\,\Phi_{kj}(k+2,r)+(10\delta_{1j}+2\delta_{23j}-2\delta_{4589j}+4\delta_{6j})\times$$
$$\times\Phi_{kj}(k+1,r)+(10\delta_{1j}+2\delta_{23457j}+6\delta_{6j})\,\Phi_{kj}(k,r)+(5\delta_{1j}-3\delta_{2j}-\delta_{3j}+$$
$$+\delta_{4j}+3\delta_{5j}+4\delta_{6j}+2\delta_{89j})\,\Phi_{kj}(|k-1|,r)+(\delta_{12569j}-\delta_{3478j})\,\Phi_{kj}\times$$
$$\times(|k-2|,r)](C_k\delta_{123j}+S_k\delta_{45j}+\widetilde{C}_k\delta_{67j}+\widetilde{S}_k\delta_{89j}),\qquad j=1,2\ldots 9.$$

For this

$$C_k=\cos(2k+1)\theta,\quad S_k=\sin(2k+1)\theta,$$
$$\widetilde{C}_k=2(2-\delta_{k0})\cos 2k\theta,\quad \widetilde{S}_k=4\sin 2k\theta,$$
$$T_{kj}=U_k\delta_{1234j}+V_k\delta_{5678j},\quad \Phi_{kj}=W_k\delta_{12345j}+Q_k\delta_{6789j},$$

and $\delta_{mn\ldots j}$ is the generalized Kronecker symbol, equal to unity when two of its arbitrary indices coincide and equal to zero in the contrary case.

The values of the functions U_k, V_k, W_k follow from the relations

$$\delta_{1l}U_k(j,r)+\delta_{2l}V_k(j,r)+\delta_{3l}W_k(j,r)=\frac{\pi}{\Gamma(1+j)}\left(\frac{z-1}{z+1}\right)^{j/2}\times$$
$$\times\left(\frac{\delta_{1l}}{2r}-\frac{r\delta_{2l}}{8b_1^2}+r\delta_{3l}\right)\sum_{m=0}^{k-1+\delta_{1l}}\frac{(-1)^m\,\Gamma(1+m-\delta_{2l})\,\Gamma(k-m+\delta_{1l})}{\Gamma(m+k+2-\delta_{1l})\,\Gamma(m-j+1-\delta_{2l})}\times$$

$$\times\left(\frac{2z}{1+z}\right)^{-m+\delta_{2l}}\left(\frac{rb_1}{2}\right)^{2m}\left[(\delta_{1l}+\delta_{3l})\cos\frac{m\pi}{2}+\delta_{2l}\sin\frac{m\pi}{2}\right]N_{1m}+$$

$$+\frac{\pi(-1)^k}{\Gamma(j+1)}\left(\frac{z-1}{z+1}\right)^{j/2}\left(\frac{\delta_{1l}}{2r}+\frac{r\delta_{2l}}{8b_1^2}-r\delta_{3l}\right)\sum_{m=0}^{\infty}\frac{\left(\frac{2z}{1+z}\right)^{-m-k-\delta_{1l}+\delta_{2l}}}{\Gamma(m+2k+2)}\times$$

$$\times\frac{\Gamma(m+k+2\delta_{1l}+\delta_{3l})\left(\frac{rb_1}{2}\right)^{2m+2k+2\delta_{1l}}}{\Gamma(m+1)\Gamma(m+k+2\delta_{1l}+\delta_{3l}-j)}N_{2m}\left\{\left[2\ln\frac{rb_1}{2}-\ln\frac{2z}{1+z}+\right.\right.$$

$$+\psi(m+k+2\delta_{1l}+\delta_{3l})-\psi(m+2k+2)-\psi(m+1)-\psi(m+k+2\delta_{1l}+$$

$$\left.\left.+\delta_{3l}-j)-H_m\right]\left[(\delta_{2l}-\delta_{1l})\sin\frac{m+k}{2}\pi+\delta_{3l}\cos\frac{m+k}{2}\pi\right]-\frac{\pi}{2}\left[(\delta_{1l}-\delta_{2l})\cos\frac{m+k}{2}\pi+\delta_{3l}\sin\frac{m+k}{2}\pi\right]\right\},$$

$$H_m=N_{2m}^{-1}\sum_{n=1}^{m+k+\delta_{1l}-\delta_{2l}}\frac{\Gamma(m+k+1+\delta_{1l}-\delta_{2l}-j)\,\Gamma(m+k+1+}{\Gamma(m+k+1+\delta_{1l}-\delta_{2l}-j-n)\,\Gamma(m+k+1+}\rightarrow$$

$$\rightarrow\frac{+\delta_{1l}-\delta_{2l})\,\Gamma(j+1)}{+\delta_{1l}-\delta_{2l}-n)\,\Gamma(n+j+1)}\frac{1}{\Gamma(n+1)}\left(\frac{z-1}{z+1}\right)^n[\psi(m+k+1+\delta_{1l}-\delta_{2l}-j-n)-\psi(m+k+1+\delta_{1l}-$$

$$-\delta_{2l}-j)+\psi(m+k+1+\delta_{1l}-\delta_{2l}-n)-\psi(m+k+1+\delta_{1l}-\delta_{2l})],$$

$$N_{1m}={}_2F_1(j-m+\delta_{2l},\ -m+\delta_{2l};\ j+1;\ (z-1)/(z+1)), \tag{7}$$

$$N_{2m}={}_2F_1(j-m-k-\delta_{1l}+\delta_{2l},-m-k-\delta_{1l}+\delta_{2l};j+1;(z-1)/(z+1)).$$

The expression $Q_k(j, r)$ is obtained from $W_k(j, r)$ by dividing the latter by r and replacing $\Gamma(m+k+2)$, $\Gamma(m+2k+2)$, and $\psi(m+2k+2)$ in it, respectively, by $\Gamma(m+k+1)$, $\Gamma(m+2k+1)$, and $\psi(m+2k+1)$.

In (7) the notation

$$b_1^2=\frac{1+\lambda}{2}b^2,\quad z=\frac{1+\lambda}{2\sqrt{\lambda}},$$

and $\psi(z)$ is the psi-function.

The fairly cumbersome solutions (6), (7) reduce to simple asymptotic expressions for $r\to 0$. Thus, with the properties of the special functions [4] just written, we find

$$\begin{gathered}A_{1,2}=\frac{1}{8\pi r}[(2\pm 1)\cos\theta\mp\cos 3\theta]+O(r),\\ A_{3,4}=\frac{1}{8\pi r}[(2\pm 1)\sin\theta\pm\sin 3\theta]+O(r),\\ A_{5,6}=\frac{(2\pm 1)r}{32b^2}\frac{1}{1+\sqrt{\lambda}}\left(1\mp\frac{1}{2\pm 1}\beta\right)\cos\theta+O(r^3),\\ A_{7,8}=\frac{(2\pm 1)r}{16b^2}\frac{1}{1+\sqrt{\lambda}}\left(1+\frac{1}{2\pm 1}\beta\right)\frac{\sin\theta}{1\mp 1+\sqrt{\lambda}\pm\sqrt{\lambda}}+O(r^3),\\ B_1=-\frac{5r}{64\pi}\alpha\cos\theta+O(r),\quad B_2=B_3=-\frac{r}{64\pi}\alpha\cos\theta+O(r),\\ B_4=B_5=-\frac{r\alpha}{64\pi}\sin\theta+O(r),\quad B_{6,7}=-\frac{2\pm 1}{32\pi}\alpha+O(1),\\ B_{8,9}=\frac{-1}{32\pi}\left(2\sin 2\theta\pm\frac{1}{2}\sin 4\theta\right)+O(r^2),\quad \alpha=2\ln r,\quad \beta=\frac{1-\sqrt{\lambda}}{1+\sqrt{\lambda}}.\end{gathered} \tag{8}$$

From the asymptotic expressions (8) and the relations (1), (2) it follows that at the point of application the tangential and shear forces t_1, t_2, t_{12} will be infinite. They have singularities of the order r^{-1}, which corresponds to the known results [6]. In addition to this, the shear force q_1 becomes infinite at $r = 0$. Its singularity has the order $\ln r$. The shear force q_2 retains a finite value, depending on the polar angle, so that on the lines of principal curvature it becomes zero. Also, the bending and twisting moments m_1, m_2, m_{12} are zero at the point under consideration.

The asymptotic expressions (8) give the values of the integrals (3) in the immediate vicinity of the point of application of the force. The following question arises: How will the solution behave when we move away from this point. From a practical viewpoint it makes sense to study the convergence of the solutions obtained for values of the argument $br/2 < 1$, since beyond the limits of these values the stress state of the shell depends on other factors, in particular, the boundary conditions, which have not been taken into account in the solution of the problem. In addition, in the case of large values of the argument the fundamental solution can be represented in another form that is more convenient for numerical realization. An analysis of the dual series (6), (7) shows that the rate of their convergence depends not only on r, but also on the ratio of the radii of principal curvature. Thus, for a spherical shell $\lambda = 1$ $(z = 1)$ in the series (6) we retain only the terms to which in (7) there corresponds $j = 0$. Since for $z = 1$, $H_m = 0$ and $N_{1m} = N_{2m} = 1$, the infinite sums (7) also are simplified and become analogous to expansions of the Thomson functions [4], which, as we know, converge very rapidly. Somewhat worse is the situation with the convergence for shells of other forms and, especially, for a cylindrical shell, when $\lambda = 0$, $(z - 1)/(z + 1) = 1$. From (8) it is seen that for $\lambda \to 0$, $r \neq 0$ the integral A_8 becomes infinite. However, this does not make the force t_{12} infinite, since $\lim\limits_{\lambda\to 0}(a_1 - \dot{a}_3)\ A_8 = 0$.

With respect to their structure, the series (6) consist of two parts. The first corresponds to a finite sum, while the second corresponds to an infinite sum with respect to m in (7). In view of the fact that the gamma function is infinite when its argument assumes a negative integer value, the first part consists of several single series with respect to k, corresponding to those values of m for which $m - j + 1 - \delta_{2l} \geq 1$. If we sum the hypergeometric functions entering there according to the expression [4]

$$ {}_2F_1(a, b; c; 1) = \frac{\Gamma(c)\,\Gamma(c - a - b)}{\Gamma(c - a)\,\Gamma(c - b)}, \tag{9} $$

then we can show that the convergence of these series will not be worse than $\sum\limits_k [\Gamma(2k - 1)]^{-1}$, i.e., will be very rapid. As for the second part, having summed by means of (9) the functions N_{2m} and H_m with the doubling expression of the gamma function [4] and the inequalities

$$ \Gamma(x + y) > \Gamma(x)\Gamma(y),\ \psi(x, y) < x + y, $$

valid for $x, y \gg 1$ taken into account, we find that the dual series for large m and k converge not worse than

$$ \sum_k \sum_m \frac{m + k}{m!\,(m + 3)!\,(k - 1)!\,\Gamma(k - 3/2)}. $$

Thus, the solutions (6), (7) remain valid for shells of both positive and zero Gaussian curvature.

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